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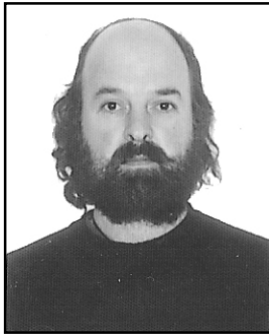
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Prof. Carlos Castro developed a keen interest in Theoretical Physics while working for his undergraduate and Ph.D theses and decided to make his carrier in teaching and research in this area. As a result he continued his research in the Extended Relativity Theory in Clifford Spaces developed by him. Gravity, Strings and Membranes, Grand-Unification; Fractals, Quantum Field Theory, Mathematical Physics, Non-commutative Geometry and Number Theory are the other areas of interest to Prof. Carlos Castro. He joined the Centre for Theoretical Studies of Physical Systems at Clark Atlanta University, Atlanta, Georgia in 1995. Since then he has been teaching and conducting research at this centre.

Earlier he taught at the University of Texas, Austin, Texas during 1984 - 1991. From September 1982 to Spring 1983, he was associated with the University of Chicago where he passed the Ph.D Candidacy Examination and worked with the Relativity Group. Before his arrival in the United States, he also studied Aeronautical Engineering at the Polytechnic of Madrid, Spain.

Prof. Carlos Castro visited India in 2003 and worked at Birla Science Centre, Hyderabad and also at the Institute of Mathematical Sciences, Chennai. He was invited to the International Centre for Theoretical Physics, Trieste, Italy from July to November 1998 and June to November 2000. In 1998 he travelled to Seoul, Korea where he worked at the Asian Pacific Center for Theoretical Physics. He was invited to the Instituto de Fisica, Guanajuato, Mexico in 1995 where he worked in the area of his interest. He was associated with the Instituto de Fisica Matematica, Sao Paulo, Brazil as well as the Centro de Investigacion y Estudios Avanzados del IPN, Mexico City during 1997. Prof. Carlos Castro became World Laboratory Fellow with Prof. George Sudarshan as Advisor at the University of Texas, Austin in 1996.

# Space-Time Compactification Induced By Lightlike Branes

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## Abstract

The aim of the present paper is two-fold. First we describe the Lagrangian dynamics of a recently proposed new class of *lightlike*  $p$ -branes and their interactions with bulk space-time gravity and electromagnetism in a self-consistent manner. Next, we discuss the role of *lightlike* branes as natural candidates for *wormhole* “throats” and exemplify the latter by presenting an explicit construction of a new type of asymmetric wormhole solution where the *lightlike* brane connects a “right” universe with Reissner-Nordström geometry to a “left” Bertotti-Robinson universe with two compactified space dimensions.

Keywords: traversable wormholes; non-Nambu-Goto lightlike branes; dynamical brane tension; black hole’s horizon “straddling”

## 1 Introduction

Lightlike branes (*LL-branes* for short) play an increasingly significant role in general relativity and modern non-perturbative string theory. Mathematically they represent singular null hypersurfaces in Riemannian space-time which provide dynamical description of various physically important cosmological and astrophysical phenomena such as:

(i) Impulsive lightlike signals arising in cataclysmic astrophysical events (supernovae, neutron star collisions) [1];

(ii) Dynamics of horizons in black hole physics – the so called “membrane paradigm” [2];

(iii) The thin-wall approach to domain walls coupled to gravity [3, 4, 5].

More recently, the relevance of *LL-branes* in the context of non-perturbative string theory has also been recognized, specifically, as the so called *H-branes* describing quantum horizons (black hole and cosmological) [6], as Penrose limits of baryonic *D-branes* [7], etc (see also Refs.[8]).

A characteristic feature of the formalism for *LL-branes* in the pioneering papers [3, 4, 5] in the context of gravity and cosmology is that they have been exclusively treated in a “phenomenological” manner, *i.e.*, without specifying an underlying Lagrangian dynamics from which they may originate. As a partial exception, in a more recent paper [9] brane actions in terms of their pertinent extrinsic geometry have been proposed which generically describe

non-lightlike branes, whereas the lightlike branes are treated as a limiting case.

On the other hand, in the last few years we have proposed in a series of papers [10, 11, 12, 13] a new class of concise manifestly reparametrization invariant world-volume Lagrangian actions, providing a derivation from first principles of the *LL-brane* dynamics. The following characteristic features of the new *LL-branes* drastically distinguish them from ordinary Nambu-Goto branes:

(a) They describe intrinsically lightlike modes, whereas Nambu-Goto branes describe massive ones.

(b) The tension of the *LL-brane* arises as an *additional dynamical degree of freedom*, whereas Nambu-Goto brane tension is a given *ad hoc* constant. The latter characteristic feature significantly distinguishes our *LL-brane* models from the previously proposed *tensionless*  $p$ -branes (for a review, see Ref.[14]). The latter rather resemble  $p$ -dimensional continuous distributions of independent massless point-particles without cohesion among the latter.

(c) Consistency of *LL-brane* dynamics in a spherically or axially symmetric gravitational background of codimension one requires the presence of an event horizon which is automatically occupied by the *LL-brane* (“horizon straddling” according to the terminology of Ref.[4]).

(d) When the *LL-brane* moves as a *test* brane in spherically or axially symmetric gravitational backgrounds its dynamical tension exhibits exponential “inflation/deflation” time behavior [11] – an effect

similar to the “mass inflation” effect around black hole horizons [15].

An intriguing novel application of *LL-branes* as natural self-consistent gravitational sources for *wormhole* space-times has been developed in a series of recent papers [12, 13, 16, 17].

Before proceeding let us recall that the concept of “wormhole space-time” was born in the classic work of Einstein and Rosen [18], where they considered matching along the horizon of two identical copies of the exterior Schwarzschild space-time region (subsequently called *Einstein-Rosen “bridge”*). Another corner stone in wormhole physics is the seminal work of Morris and Thorne [19], who studied for the first time *traversable Lorentzian wormholes*.

In what follows, when discussing wormholes we will have in mind the physically important class of “thin-shell” traversable Lorentzian wormholes first introduced by Visser [20, 21]. For a comprehensive review of wormhole space-times, see Refs.[21, 22].

In our earlier work [12, 13, 16, 17] we have constructed various types of wormhole solutions in self-consistent systems of bulk gravity and bulk gauge fields (Maxwell and Kalb-Ramond) coupled to *LL-branes* where the latter provide the appropriate stress energy tensors, electric currents and dynamically generated space-varying cosmological constant terms consistently derived from well-defined world-volume *LL-brane* Lagrangian actions.

The original Einstein-Rosen “bridge” manifold [18] appears as a particular case of the construction of spherically symmetric wormholes produced by *LL-branes* as gravitational sources occupying the wormhole throats (Refs.[16, 13]). Thus, we are lead to the important conclusion that consistency of Einstein equations of motion yielding the original Einstein-Rosen “bridge” as well-defined solution necessarily requires the presence of *LL-brane* energy-momentum tensor as a source on the right hand side.

More complicated examples of spherically and axially symmetric wormholes with Reissner-Nordström and rotating cylindrical geometry, respectively, have been explicitly constructed via *LL-branes* in Refs.[12, 13]. Namely, two copies of the exterior space-time region of a Reissner-Nordström or rotating cylindrical black hole, respectively, are matched via *LL-brane* along what used to be the outer horizon of the respective full black hole space-time manifold. In this way one obtains a wormhole solution which combines the features of the Einstein-Rosen “bridge” on the one hand (with wormhole throat at horizon), and the features of Misner-Wheeler wormholes [23], *i.e.*, exhibiting the so called “charge without charge” phe-

nomenon.

Recently the results of Refs.[12, 13] have been extended to the case of *asymmetric* wormholes, describing two “universes” with different spherically symmetric geometries of black hole type connected via a “throat” materialized by the pertinent gravitational source – an electrically charged *LL-brane*, sitting on their common horizon. As a result of the well-defined world-volume *LL-brane* dynamics coupled self-consistently to gravity and bulk space-time gauge fields, it creates a “left universe” comprising the exterior Schwarzschild-de-Sitter space-time region beyond the Schwarzschild horizon and where the cosmological constant is dynamically generated, and a “right universe” comprising the exterior Reissner-Nordström region beyond the outer Reissner-Nordström horizon with dynamically generated Coulomb field-strength. Both “universes” are glued together by the *LL-brane* occupying their common horizon. Similarly, the *LL-brane* can dynamically generate a non-zero cosmological constant in the “right universe”, in which case it connects a purely Schwarzschild “left universe” with a Reissner-Nordström-de-Sitter “right universe”.

In the present paper we will further broaden the application of *LL-branes* in the context of wormhole physics by constructing a new type of wormhole solution to Einstein-Maxwell equations describing a “right universe”, which comprises the exterior Reissner-Nordström space-time region beyond the outer Reissner-Nordström horizon, connected through a “throat” materialized by a *LL-brane* with a “left universe” being a Bertotti-Robinson space-time with two compactified spatial dimensions [24] (see also [25]).

Let us note that previously the junction of a compactified space-time (of Bertotti-Robinson type) to an uncompactified space-time through a wormhole has been studied in a different setting using *time-like* matter on the junction hypersurface [26]. Also, in a different context a string-like (flux tube) object with similar features to Bertotti-Robinson solution has been constructed [27] which interpolates between uncompactified space-time regions.

## 2 World-Volume Formulation of Lightlike Brane Dynamics

There exist two equivalent *dual to each other* manifestly reparametrization invariant world-volume Lagrangian formulations of *LL-branes* [10, 11, 12, 13, 16, 28]. First, let us consider the Polyakov-type for-

mulation where the  $LL$ -brane world-volume action is given as:

$$S_{LL} = \int d^{p+1}\sigma \Phi \left[ -\frac{1}{2}\gamma^{ab}g_{ab} + L(F^2) \right]. \quad (1)$$

Here the following notions and notations are used:

(a)  $\Phi$  is alternative non-Riemannian integration measure density (volume form) on the  $p$ -brane world-volume manifold:

$$\Phi \equiv \frac{1}{(p+1)!} \varepsilon^{a_1 \dots a_{p+1}} H_{a_1 \dots a_{p+1}}(B), \quad (2)$$

$$H_{a_1 \dots a_{p+1}}(B) = (p+1) \partial_{[a_1} B_{a_2 \dots a_{p+1}]}, \quad (3)$$

instead of the usual  $\sqrt{-\gamma}$ . Here  $\varepsilon^{a_1 \dots a_{p+1}}$  is the alternating symbol ( $\varepsilon^{01 \dots p} = 1$ ),  $\gamma_{ab}$  ( $a, b = 0, 1, \dots, p$ ) indicates the intrinsic Riemannian metric on the world-volume, and  $\gamma = \det \|\gamma_{ab}\|$ .  $H_{a_1 \dots a_{p+1}}(B)$  denotes the field-strength of an auxiliary world-volume antisymmetric tensor gauge field  $B_{a_1 \dots a_p}$  of rank  $p$ . As a special case one can build  $H_{a_1 \dots a_{p+1}}$  in terms of  $p+1$  auxiliary world-volume scalar fields  $\{\varphi^I\}_{I=1}^{p+1}$ :

$$H_{a_1 \dots a_{p+1}} = \varepsilon_{I_1 \dots I_{p+1}} \partial_{a_1} \varphi^{I_1} \dots \partial_{a_{p+1}} \varphi^{I_{p+1}}. \quad (4)$$

Note that  $\gamma_{ab}$  is *independent* of the auxiliary world-volume fields  $B_{a_1 \dots a_p}$  or  $\varphi^I$ . The alternative non-Riemannian volume form (2) has been first introduced in the context of modified standard (non-lightlike) string and  $p$ -brane models in Refs.[29].

(b)  $X^\mu(\sigma)$  are the  $p$ -brane embedding coordinates in the bulk  $D$ -dimensional space time with bulk Riemannian metric  $G_{\mu\nu}(X)$  with  $\mu, \nu = 0, 1, \dots, D-1$ ; ( $\sigma \equiv (\sigma^0 \equiv \tau, \sigma^i)$  with  $i = 1, \dots, p$ ;  $\partial_a \equiv \frac{\partial}{\partial \sigma^a}$ ).

(c)  $g_{ab}$  is the induced metric on world-volume:

$$g_{ab} \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X), \quad (5)$$

which becomes *singular* on-shell (manifestation of the lightlike nature, cf. second Eq.(10) below).

(d)  $L(F^2)$  is the Lagrangian density of another auxiliary  $(p-1)$ -rank antisymmetric tensor gauge field  $A_{a_1 \dots a_{p-1}}$  on the world-volume with  $p$ -rank field-strength and its dual:

$$F_{a_1 \dots a_p} = p \partial_{[a_1} A_{a_2 \dots a_p]}, \quad F^{*a} = \frac{1}{p!} \frac{\varepsilon^{a a_1 \dots a_p}}{\sqrt{-\gamma}} F_{a_1 \dots a_p}. \quad (6)$$

$L(F^2)$  is *arbitrary* function of  $F^2$  with the short-hand notation:  $F^2 \equiv F_{a_1 \dots a_p} F_{b_1 \dots b_p} \gamma^{a_1 b_1} \dots \gamma^{a_p b_p}$ .

Rewriting the action (1) in the following equivalent form:

$$S = - \int d^{p+1}\sigma \chi \sqrt{-\gamma} \left[ \frac{1}{2} \gamma^{ab} g_{ab} - L(F^2) \right], \quad (7)$$

$$\chi \equiv \frac{\Phi}{\sqrt{-\gamma}}$$

with  $\Phi$  the same as in (2), we find that the composite field  $\chi$  plays the role of a *dynamical (variable) brane tension*<sup>1</sup>.

Let us now consider the equations of motion corresponding to (1) w.r.t.  $B_{a_1 \dots a_p}$ :

$$\partial_a \left[ \frac{1}{2} \gamma^{cd} g_{cd} - L(F^2) \right] = 0 \quad \rightarrow \quad \frac{1}{2} \gamma^{cd} g_{cd} - L(F^2) = M, \quad (8)$$

where  $M$  is an arbitrary integration constant. The equations of motion w.r.t.  $\gamma^{ab}$  read:

$$\frac{1}{2} g_{ab} - F^2 L'(F^2) \left[ \gamma_{ab} - \frac{F_a^* F_b^*}{F^{*2}} \right] = 0, \quad (9)$$

where  $F^{*a}$  is the dual field strength (6). Eqs.(9) can be viewed as  $p$ -brane analogues of the string Virasoro constraints.

Taking the trace in (9) and comparing with (8) implies the following crucial relation for the Lagrangian function  $L(F^2)$ :  $L(F^2) - p F^2 L'(F^2) + M = 0$ , which determines  $F^2$  on-shell as certain function of the integration constant  $M$  (8), i.e.  $F^2 = F^2(M) = \text{const}$ . Here and below  $L'(F^2)$  denotes derivative of  $L(F^2)$  w.r.t. the argument  $F^2$ .

The next and most profound consequence of Eqs.(9) is that the induced metric (5) on the world-volume of the  $p$ -brane model (1) is *singular* on-shell (as opposed to the induced metric in the case of ordinary Nambu-Goto branes):

$$g_{ab} F^{*b} \equiv \partial_a X^\mu G_{\mu\nu} (\partial_b X^\nu F^{*b}) = 0. \quad (10)$$

Eq.(10) is the manifestation of the *lightlike* nature of the  $p$ -brane model (1) (or (7)), namely, the tangent vector to the world-volume  $F^{*a} \partial_a X^\mu$  is *lightlike* w.r.t. metric of the embedding space-time.

Further, the equations of motion w.r.t. world-volume gauge field  $A_{a_1 \dots a_{p-1}}$  (with  $\chi$  as defined in (7) read:

$$\partial_{[a} (F_{b]}^* \chi) = 0. \quad (11)$$

Finally, the  $X^\mu$  equations of motion produced by the (1) read:

$$\partial_a (\chi \sqrt{-\gamma} \gamma^{ab} \partial_b X^\mu) + \chi \sqrt{-\gamma} \gamma^{ab} \partial_a X^\nu \partial_b X^\lambda \Gamma_{\nu\lambda}^\mu = 0 \quad (12)$$

where  $\Gamma_{\nu\lambda}^\mu = \frac{1}{2} G^{\mu\kappa} (\partial_\nu G_{\kappa\lambda} + \partial_\lambda G_{\kappa\nu} - \partial_\kappa G_{\nu\lambda})$  is the Christoffel connection for the external metric.

<sup>1</sup>The notion of dynamical brane tension has previously appeared in different contexts in Refs.[30].

Eq.(11) allows us to introduce the dual “gauge” potential  $u$  (dual w.r.t. world-volume gauge field  $A_{a_1 \dots a_{p-1}}$  (6)) :

$$F_a^* = c_p \frac{1}{\chi} \partial_a u \quad , \quad c_p = \text{const} . \quad (13)$$

Relation (13) enables us to rewrite Eq.(9) (the light-like constraint) in terms of the dual potential  $u$  in the form:

$$\begin{aligned} \gamma_{ab} &= \frac{1}{2a_0} g_{ab} - \frac{(2a_0)^{p-2}}{\chi^2} \partial_a u \partial_b u \\ a_0 &\equiv F^2 L'(F^2) \Big|_{F^2=F^2(M)} = \text{const} . \end{aligned} \quad (14)$$

( $L'(F^2)$  denotes derivative of  $L(F^2)$  w.r.t. the argument  $F^2$ ). From (13) we obtain the relation:

$$\chi^2 = -(2a_0)^{p-2} \gamma^{ab} \partial_a u \partial_b u , \quad (15)$$

and the Bianchi identity  $\nabla_a F^{*a} = 0$  becomes:

$$\partial_a \left( \frac{1}{\chi} \sqrt{-\gamma} \gamma^{ab} \partial_b u \right) = 0 . \quad (16)$$

It is straightforward to prove that the system of equations (12), (16) and (15) for  $(X^\mu, u, \chi)$ , which are equivalent to the equations of motion (8)–(11),(12) resulting from the original Polyakov-type *LL-brane* action (1), can be equivalently derived from the following *dual* Nambu-Goto-type world-volume action:

$$S_{\text{NG}} = - \int d^{p+1} \sigma T \sqrt{\left| \det \|g_{ab} - \epsilon \frac{1}{T^2} \partial_a u \partial_b u\| \right|} , \quad (17)$$

with  $\epsilon = \pm 1$ . Here again  $g_{ab}$  indicates the induced metric on the world-volume (5) and  $T$  is dynamical variable tension simply proportional to  $\chi$  ( $\chi^2 = (2a_0)^{p-1} T^2$  with  $a_0$  as in (14)). The choice of the sign in (17) does not have physical effect because of the non-dynamical nature of the  $u$ -field.

Henceforth we will stick to the Polyakov-type formulation of world-volume *LL-brane* dynamics since within this framework one can add in a natural way [10, 11, 12] couplings of the *LL-brane* to bulk space-time Maxwell  $\mathcal{A}_\mu$  and Kalb-Ramond  $\mathcal{A}_{\mu_1 \dots \mu_{D-1}}$  gauge fields (in the case of codimension one *LL-branes*, i.e., for  $D = (p+1) + 1$ ):

$$\begin{aligned} \tilde{S}_{\text{LL}} &= S_{\text{LL}} - q \int d^{p+1} \sigma \varepsilon^{a b_1 \dots b_p} F_{b_1 \dots b_p} \partial_a X^\mu \mathcal{A}_\mu \\ &- \frac{\beta}{(p+1)!} \int d^{p+1} \sigma \varepsilon^{a_1 \dots a_{p+1}} \partial_{a_1} X^{\mu_1} \dots \partial_{a_{p+1}} X^{\mu_{p+1}} \\ &\quad \times \mathcal{A}_{\mu_1 \dots \mu_{p+1}} \end{aligned} \quad (18)$$

with  $S_{\text{LL}}$  as in (1). The *LL-brane* constraint equations (8)–(9) are not affected by the bulk space-time gauge field couplings whereas Eqs.(11)–(12) acquire the form:

$$\begin{aligned} \partial_{[a} \left( F_{b]}^* \chi L'(F^2) \right) &+ \frac{q}{4} \partial_a X^\mu \partial_b X^\nu \mathcal{F}_{\mu\nu} = 0 ; \quad (19) \\ \partial_a \left( \chi \sqrt{-\gamma} \gamma^{ab} \partial_b X^\mu \right) &+ \chi \sqrt{-\gamma} \gamma^{ab} \partial_a X^\nu \partial_b X^\lambda \Gamma_{\nu\lambda}^\mu \\ &- q \varepsilon^{a b_1 \dots b_p} F_{b_1 \dots b_p} \partial_a X^\nu \mathcal{F}_{\lambda\nu} G^{\lambda\mu} \\ &- \frac{\beta}{(p+1)!} \varepsilon^{a_1 \dots a_{p+1}} \partial_{a_1} X^{\mu_1} \dots \partial_{a_{p+1}} X^{\mu_{p+1}} \\ &\quad \times \mathcal{F}_{\lambda\mu_1 \dots \mu_{p+1}} G^{\lambda\mu} = 0 . \end{aligned} \quad (20)$$

Here  $\chi$  is the dynamical brane tension as in (7),  $\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$  and

$$\mathcal{F}_{\mu_1 \dots \mu_D} = D \partial_{[\mu_1} \mathcal{A}_{\mu_2 \dots \mu_D]} = \mathcal{F} \sqrt{-G} \varepsilon_{\mu_1 \dots \mu_D} \quad (21)$$

are the field-strengths of the electromagnetic  $\mathcal{A}_\mu$  and Kalb-Ramond  $\mathcal{A}_{\mu_1 \dots \mu_{D-1}}$  gauge potentials [31].

### 3 Lightlike Brane Dynamics in Various Types of Gravitational Backgrounds

World-volume reparametrization invariance allows us to introduce the standard synchronous gauge-fixing conditions:

$$\gamma^{0i} = 0 \quad (i = 1, \dots, p) , \quad \gamma^{00} = -1 . \quad (22)$$

Also, we will use a natural ansatz for the “electric” part of the auxiliary world-volume gauge field-strength (6):

$$F^{*i} = 0 \quad (i = 1, \dots, p) \quad , \quad \text{i.e.} \quad F_{0i_1 \dots i_{p-1}} = 0 , \quad (23)$$

meaning that we choose the lightlike direction in Eq.(10) to coincide with the brane proper-time direction on the world-volume ( $F^{*a} \partial_a \sim \partial_\tau$ ). The Bianchi identity ( $\nabla_a F^{*a} = 0$ ) together with (22)–(23) and the definition for the dual field-strength in (6) imply:

$$\partial_\tau \gamma^{(p)} = 0 \quad \text{where} \quad \gamma^{(p)} \equiv \det \|\gamma_{ij}\| . \quad (24)$$

Taking into account (22)–(23), Eqs.(9) acquire the following gauge-fixed form (recall definition of the induced metric  $g_{ab}$  (5)):

$$g_{00} \equiv \dot{X}^\mu G_{\mu\nu} \dot{X}^\nu = 0 \quad , \quad g_{0i} = 0 \quad , \quad g_{ij} - 2a_0 \gamma_{ij} = 0 , \quad (25)$$

where  $a_0$  is the same constant as in (14).

### 3.1 Spherically Symmetric Backgrounds

Here we will be interested in static spherically symmetric solutions of Einstein-Maxwell equations (see Eqs.(35)–(36) below). We will consider the following generic form of static spherically symmetric metric:

$$ds^2 = -A(\eta)dt^2 + \frac{d\eta^2}{A(\eta)} + C(\eta)h_{ij}(\vec{\theta})d\theta^i d\theta^j, \quad (26)$$

or, in Eddington-Finkelstein coordinates [32] ( $dt = dv - \frac{d\eta}{A(\eta)}$ ):

$$ds^2 = -A(\eta)dv^2 + 2dv d\eta + C(\eta)h_{ij}(\vec{\theta})d\theta^i d\theta^j. \quad (27)$$

Here  $h_{ij}$  indicates the standard metric on the sphere  $S^p$ . The radial-like coordinate  $\eta$  will vary in general from  $-\infty$  to  $+\infty$ .

We will consider the simplest ansatz for the *LL-brane* embedding coordinates:

$$\begin{aligned} X^0 \equiv v = \tau, \quad X^1 \equiv \eta = \eta(\tau) \\ X^i \equiv \theta^i = \sigma^i \quad (i = 1, \dots, p). \end{aligned} \quad (28)$$

Now, the *LL-brane* equations (25) together with (24) yield:

$$-A(\eta) + 2\dot{\eta} = 0, \quad \partial_\tau C = \dot{\eta} \partial_\eta C \Big|_{\eta=\eta(\tau)} = 0. \quad (29)$$

First, we will consider the case of  $C(\eta)$  as non-trivial function of  $\eta$  (i.e., proper spherically symmetric space-time). In this case Eqs.(29) imply:

$$\dot{\eta} = 0 \rightarrow \eta(\tau) = \eta_0 = \text{const}, \quad A(\eta_0) = 0. \quad (30)$$

Eq.(30) tells us that consistency of *LL-brane* dynamics in a proper spherically symmetric gravitational background of codimension one requires the latter to possess a horizon (at some  $\eta = \eta_0$ ), which is automatically occupied by the *LL-brane* (“horizon straddling” according to the terminology of Ref.[4]). Similar property – “horizon straddling”, has been found also for *LL-branes* moving in rotating axially symmetric (Kerr or Kerr-Newman) and rotating cylindrically symmetric black hole backgrounds [12, 13].

With the embedding ansatz (28) and assuming the bulk Maxwell field to be purely electric static one ( $\mathcal{F}_{0\eta} = \mathcal{F}_{v\eta} \neq 0$ , the rest being zero; this is the relevant case to be discussed in what follows), Eq.(19) yields the simple relation:  $\partial_i \chi = 0$ , i.e.  $\chi = \chi(\tau)$ . Further, the only non-trivial contribution of the second order (w.r.t. world-volume proper time derivative)  $X^\mu$ -equations of motion (20) arises for  $\mu = v$ , where the latter takes the form of an evolution equation for the dynamical tension  $\chi(\tau)$ . In the case of absence of couplings to bulk space-time gauge fields, the

latter yields exponential “inflation”/“deflation” at large times for the dynamical *LL-brane* tension:

$$\chi(\tau) = \chi_0 \exp\left\{-\tau \left(\frac{1}{2} \partial_\eta A + p a_0 \partial_\eta C\right)_{\eta=\eta_0}\right\}, \quad (31)$$

$\chi_0 = \text{const}$ . Similarly to the “horizon straddling” property, exponential “inflation”/“deflation” for the *LL-brane* tension has also been found in the case of test *LL-brane* motion in rotating axially symmetric and rotating cylindrically symmetric black hole backgrounds (for details we refer to Refs.[11, 12, 13]). This phenomenon is an analog of the “mass inflation” effect around black hole horizons [15].

### 3.2 Product-Type Gravitational Backgrounds: Bertotti-Robinson Space-Time

Consider now the case  $C(\eta) = \text{const}$  in (27), i.e., the corresponding space-time manifold is of product type  $\Sigma_2 \times S^p$ . A physically relevant example is the Bertotti-Robinson [24, 25] space-time in  $D = 4$  (i.e.,  $p = 2$ ) with (non-extremal) metric (cf.[25]):

$$ds^2 = r_0^2 \left[ -\eta^2 dt^2 + \frac{d\eta^2}{\eta^2} + d\theta^2 + \sin^2 \theta d\varphi^2 \right], \quad (32)$$

or in Eddington-Finkelstein (EF) form ( $dt = \frac{1}{r_0} dv - \frac{d\eta}{\eta^2}$ ):

$$ds^2 = -\frac{\eta^2}{r_0^2} dv^2 + 2dv d\eta + r_0^2 [d\theta^2 + \sin^2 \theta d\varphi^2]. \quad (33)$$

At  $\eta = 0$  the Bertotti-Robinson metric (32) (or (33)) possesses a horizon. Further, we will consider the case of Bertotti-Robinson universe with constant electric field  $\mathcal{F}_{v\eta} = \pm \frac{1}{2r_0\sqrt{\pi}}$ . In the present case the second Eq.(29) is trivially satisfied whereas the first one yields:  $\eta(\tau) = \eta(0) \left(1 - \tau \frac{\eta(0)}{2r_0^2}\right)^{-1}$ . In particular, if the *LL-brane* is initially (at  $\tau = 0$ ) located on the Bertotti-Robinson horizon  $\eta = 0$ , it will stay there permanently.

## 4 Self-Consistent Wormhole Solutions Produced By Lightlike Branes

### 4.1 Lagrangian Formulation of Bulk Gravity-Matter System Coupled to Lightlike Brane

Let us now consider self-consistent bulk Einstein-Maxwell-Kalb-Ramond system coupled to a charged



codimension-one *lightlike*  $p$ -brane (i.e.,  $D = (p + 1) + 1$ ). It is described by the following action:

$$S = \int d^D x \sqrt{-G} \left[ \frac{R(G)}{16\pi} - \frac{1}{4} \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} - \frac{1}{D!2} \mathcal{F}_{\mu_1 \dots \mu_D} \mathcal{F}^{\mu_1 \dots \mu_D} \right] + \tilde{S}_{LL}. \quad (34)$$

Here  $\mathcal{F}_{\mu\nu}$  and  $\mathcal{F}_{\mu_1 \dots \mu_D}$  are the Maxwell and Kalb-Ramond field-strengths (21). The last term on the r.h.s. of (34) indicates the reparametrization invariant world-volume action (18) of the *LL-brane* coupled to the bulk space-time gauge fields.

The pertinent Einstein-Maxwell-Kalb-Ramond equations of motion derived from the action (34) read:

$$R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} R = 8\pi \left( T_{\mu\nu}^{(EM)} + T_{\mu\nu}^{(KR)} + T_{\mu\nu}^{(brane)} \right), \quad (35)$$

$$\partial_\nu \left( \sqrt{-G} \mathcal{F}^{\mu\nu} \right) + q \int d^{p+1} \sigma \delta^{(D)}(x - X(\sigma)) \times \varepsilon^{ab_1 \dots b_p} F_{b_1 \dots b_p} \partial_a X^\mu = 0, \quad (36)$$

$$\varepsilon^{\nu\mu_1 \dots \mu_{p+1}} \partial_\nu \mathcal{F} - \beta \int d^{p+1} \sigma \delta^{(D)}(x - X(\sigma)) \times \varepsilon^{a_1 \dots a_{p+1}} \partial_{a_1} X^{\mu_1} \dots \partial_{a_{p+1}} X^{\mu_{p+1}} = 0, \quad (37)$$

where in the last equation we have used the last relation (21). The explicit form of the energy-momentum tensors read:

$$T_{\mu\nu}^{(EM)} = \mathcal{F}_{\mu\kappa} \mathcal{F}_{\nu\lambda} G^{\kappa\lambda} - G_{\mu\nu} \frac{1}{4} \mathcal{F}_{\rho\kappa} \mathcal{F}_{\sigma\lambda} G^{\rho\sigma} G^{\kappa\lambda}, \quad (38)$$

$$T_{\mu\nu}^{(KR)} = \frac{1}{(D-1)!} \left[ \mathcal{F}_{\mu\lambda_1 \dots \lambda_{D-1}} \mathcal{F}_{\nu}^{\lambda_1 \dots \lambda_{D-1}} - \frac{1}{2D} G_{\mu\nu} \mathcal{F}_{\lambda_1 \dots \lambda_D} \mathcal{F}^{\lambda_1 \dots \lambda_D} \right] = -\frac{1}{2} \mathcal{F}^2 G_{\mu\nu}, \quad (39)$$

$$T_{\mu\nu}^{(brane)} = -G_{\mu\kappa} G_{\nu\lambda} \int d^{p+1} \sigma \frac{\delta^{(D)}(x - X(\sigma))}{\sqrt{-G}} \times \chi \sqrt{-\gamma} \gamma^{ab} \partial_a X^\kappa \partial_b X^\lambda, \quad (40)$$

where the brane stress-energy tensor is straightforwardly derived from the world-volume action (1) (or, equivalently, (7); recall  $\chi \equiv \frac{\Phi}{\sqrt{-\gamma}}$  is the variable brane tension).

Using again the embedding ansatz (28) together with (30) as well as (22)–(25), the Kalb-Ramond equations of motion (37) reduce to:

$$\partial_\eta \mathcal{F} + \beta \delta(\eta - \eta_0) = 0 \quad (41)$$

implying

$$\begin{aligned} \mathcal{F} &= \mathcal{F}_{(+)} \theta(\eta - \eta_0) + \mathcal{F}_{(-)} \theta(\eta_0 - \eta) \\ \mathcal{F}_{(\pm)} &= \text{const} \quad , \quad \mathcal{F}_{(-)} - \mathcal{F}_{(+)} = \beta \end{aligned} \quad (42)$$

Therefore, a space-time varying non-negative cosmological constant is dynamically generated in both exterior and interior regions w.r.t. the horizon at  $\eta = \eta_0$  (cf. Eq.(39)):  $\Lambda_{(\pm)} = 4\pi \mathcal{F}_{(\pm)}^2$ . Hereafter we will discard the presence of the Kalb-Ramond gauge field and, correspondingly, there will be no dynamical generation of cosmological constant.

## 4.2 Asymmetric Wormholes

We will consider in what follows the case of  $D = 4$  dimensional bulk space-time and, correspondingly,  $p = 2$  for the *LL-brane*. For further simplification of the numerical constant factors we will choose the following specific (“wrong-sign” Maxwell) form for the Lagrangian of the auxiliary non-dynamical world-volume gauge field (6):  $L(F^2) = \frac{1}{4} F^2 \rightarrow a_0 = M$ , where again  $a_0$  is the constant defined in (14) and  $M$  denotes the original integration constant in Eqs.(8).

We will seek a self-consistent solution of the equations of motion of the coupled Einstein-Maxwell-*LL-brane* system (Eqs.(35)–(36) and (8)–(9), (19)–(20)) describing an asymmetric wormhole space-time with spherically symmetric geometry. The general form of asymmetric wormhole metric (in Eddington-Finkelstein coordinates) reads:

$$ds^2 = -A(\eta) dv^2 + 2dv d\eta + C(\eta) [d\theta^2 + \sin^2 \theta d\varphi^2], \quad (43)$$

$$A(0) = 0 \quad (\text{“throat” at } \eta_0 = 0)$$

$$A(\eta) > 0 \quad \text{for all } \eta \neq 0. \quad (44)$$

The radial-like coordinate  $\eta$  varies from  $-\infty$  to  $+\infty$  and the metric coefficients  $A(\eta)$  and  $C(\eta)$  are continuous but *not necessarily differentiable* w.r.t.  $\eta$  at the wormhole “throat”  $\eta = 0$ . We will require:

$$\partial_\eta A \Big|_{\eta \rightarrow +0} \equiv \partial_\eta A \Big|_{+0} > 0, \quad \partial_\eta A \Big|_{\eta \rightarrow -0} \equiv \partial_\eta A \Big|_{-0} > 0. \quad (45)$$

Einstein equations (35) yield for the metric (43):

$$\begin{aligned} \partial_\eta A \Big|_{+0} - \partial_\eta A \Big|_{-0} &= -16\pi \chi \\ \partial_\eta \ln C \Big|_{+0} - \partial_\eta \ln C \Big|_{-0} &= -\frac{4\pi \chi}{a_0}. \end{aligned} \quad (46)$$

For the *LL-brane* equations of motion we use again the embedding (28) resulting in the *LL-brane* “horizon straddling” (30). On the other hand, the second order Eqs.(20) contain “force” terms (the geodesic ones involving the Christoffel connection coefficients as well as those coming from the *LL-brane* coupling to the bulk Maxwell gauge field) which display discontinuities across the “throat” at  $\eta = 0$  occupied by the *LL-brane* due to the delta-function terms

in the respective bulk space-time Einstein-Maxwell Eqs.(35)–(36) (now  $\eta_0 \equiv 0$ ). The discontinuity problem is resolved following the approach in Ref.[3] (see also the regularization approach in Ref.[33], Appendix A) by taking mean values of the “force” terms across the discontinuity at  $\eta = 0$ . Furthermore, we will require  $\chi = \text{const}$  (independent of the *LL-brane* proper time  $\tau$ ) for consistency with the matching relations (46). Thus, in the case of the *LL-brane* embedding (28) the  $X^\mu$ -equation (20) for  $\mu = v$  with  $D = 4$ ,  $p = 2$ , no Kalb-Ramond coupling, *i.e.*,  $\mathcal{F} = 0$ , and using the gauge-fixing (22), becomes:

$$\chi \left[ \frac{1}{4} \left( \partial_\eta A \Big|_{+0} + \partial_\eta A \Big|_{-0} \right) + a_0 \left( \partial_\eta \ln C \Big|_{+0} + \partial_\eta \ln C \Big|_{-0} \right) \right] - q\sqrt{2a_0} \left[ \mathcal{F}_{v\eta} \Big|_{+0} + \mathcal{F}_{v\eta} \Big|_{-0} \right] = 0 \quad (47)$$

In the present wormhole solution we will take “left” Bertotti-Robinson “universe” with:

$$A(\eta) = \frac{\eta^2}{r_0^2}, \quad C(\eta) = r_0^2, \quad \mathcal{F}_{v\eta} = \pm \frac{1}{2\sqrt{\pi} r_0} \quad (48)$$

for  $\eta < 0$ , and “right” Reissner-Nordström “universe” with:

$$A(\eta) \equiv A_{\text{RN}}(r_0 + \eta) = 1 - \frac{2m}{r_0 + \eta} + \frac{Q^2}{(r_0 + \eta)^2},$$

$$C(\eta) = (r_0 + \eta)^2, \quad \mathcal{F}_{v\eta} \equiv \mathcal{F}_{vr} \Big|_{\text{RN}} = \frac{Q}{\sqrt{4\pi}(r_0 + \eta)^2}, \quad (49)$$

for  $\eta > 0$ , and

$$A(0) \equiv A_{\text{RN}}(r_0) = 0, \quad \partial_\eta A \Big|_{+0} \equiv \partial_r A_{\text{RN}} \Big|_{r=r_0} > 0 \quad (50)$$

where  $\mathcal{F}_{v\eta}$ 's are the respective Maxwell field-strengths and where  $Q = r_0 \left[ \sqrt{\frac{8\pi}{a_0}} q r_0 \pm 1 \right]$  is determined from the discontinuity of  $\mathcal{F}_{v\eta}$  in Maxwell equations (36) across the charged *LL-brane*. Here we have used the standard coordinate notations for the Reissner-Nordström metric coefficients and Coulomb field strength:

$$A_{\text{RN}}(r) = 1 - \frac{2m}{r} + \frac{Q^2}{r^2}, \quad \mathcal{F}_{vr} \Big|_{\text{RN}} = \frac{Q}{\sqrt{4\pi} r^2}. \quad (51)$$

Since obviously both Bertotti-Robinson (48) and Reissner-Nordström (49) metrics do satisfy the “vacuum” Einstein-Maxwell equations (Eqs.(35)–(36) *without* the *LL-brane* stress-energy tensor) it remains to check the matching of both metrics at the “throat”  $\eta = 0$  (the location of the *LL-brane*) according to

Eqs.(46)–(47). In this case the latter equations give:

$$\partial_r A_{\text{RN}} \Big|_{r=r_0} = -16\pi \chi, \quad \partial_r \ln r^2 \Big|_{r=r_0} = -\frac{4\pi}{a_0} \chi \quad (52)$$

$$\chi \left[ \frac{1}{4} \partial_r A_{\text{RN}} \Big|_{r=r_0} + a_0 \partial_r \ln r^2 \Big|_{r=r_0} \right] - 2q^2 \mp \frac{q}{r_0} \sqrt{\frac{2a_0}{\pi}} = 0. \quad (53)$$

From (52)–(53) we get:

$$r_0 = \frac{a_0}{2\pi|\chi|}, \quad m = \frac{a_0}{2\pi|\chi|} (1 - 4a_0), \quad (54)$$

implying that the dynamical *LL-brane* tension  $\chi$  must be negative, thus identifying the *LL-brane* as “exotic matter” [19, 21]. Further we obtain a quadratic equation for  $|\chi|$ :

$$\chi^2 + \frac{q^2}{4\pi} \pm \frac{q}{2\sqrt{2\pi} a_0} |\chi| = 0, \quad (55)$$

which dictates that we have to choose the sign of  $q$  to be opposite to the sign in the expression for the Bertotti-Robinson constant electric field (last Eq.(48)). There are two positive solutions for  $|\chi|$ :

$$|\chi| = \frac{|q|}{4\sqrt{2\pi} a_0} (1 \pm \sqrt{1 - 8a_0}) \quad \text{for } a_0 < 1/8. \quad (56)$$

Using (54) and (56) the expression for  $Q^2$  reads:

$$Q^2 = \frac{a_0^2}{4\pi^2 \chi^2} (1 - 8a_0) = \frac{8a_0^3}{\pi q^2} \frac{1 - 8a_0}{(1 \pm \sqrt{1 - 8a_0})^2} \quad (57)$$

Thus, we have constructed a solution to Einstein-Maxwell equations (35)–(36) in  $D = 4$  describing a wormhole space-time manifold consisting of a “left” Bertotti-Robinson universe with two compactified space dimensions and a “right” Reissner-Nordström universe connected by a “throat” materialized by a *LL-brane*. The “throat” is a common horizon for both universes where for the “right” universe it is the external Reissner-Nordström horizon. All wormhole parameters, including the dynamical *LL-brane* tension, are determined in terms of the surface charge density  $q$  of the *LL-brane* (cf. Eq.(18)) and the integration constant  $a_0$  (14) characterizing *LL-brane* dynamics in a bulk gravitational field.

## 5 Conclusions. Travel to Compactland Through a Wormhole

In this work we have explored the use of (codimension-one) *LL-branes* for construction of new

asymmetric wormhole solutions of Einstein-Maxwell equations. We have put strong emphasize on the crucial properties of the dynamics of *LL-branes* interacting with gravity and bulk space-time gauge fields:

(i) “Horizon straddling” – automatic position of the *LL-brane* on (one of) the horizon(s) of the bulk space-time geometry;

(ii) Intrinsic nature of the *LL-brane* tension as an additional *dynamical degree of freedom* unlike the case of standard Nambu-Goto *p*-branes;

(iii) The *LL-brane* stress-energy tensor is systematically derived from the underlying *LL-brane* Lagrangian action and provides the appropriate source term on the r.h.s. of Einstein equations to enable the existence of consistent non-trivial wormhole solutions;

(iv) Electrically charged *LL-branes* naturally produce *asymmetric* wormholes with the *LL-brane* itself materializing the wormhole “throat” and uniquely determining the pertinent wormhole parameters.

Finally, let us point out that the above asymmetric wormhole connecting Reissner-Nordström universe with a Bertotti-Robinson universe through a lightlike hypersurface occupied by a *LL-brane* is *traversable* w.r.t. the *proper time* of a traveling observer. The latter property is similar to the *proper time* traversability of other symmetric and asymmetric wormholes with *LL-brane* sitting on the “throat” [12, 13, 16, 17]. Indeed, let us consider test particle (“traveling observer”) dynamics in the asymmetric wormhole background given by (48)–(49), which is described by the action:

$$S_{\text{particle}} = \frac{1}{2} \int d\lambda \left[ \frac{1}{e} \dot{x}^\mu \dot{x}^\nu G_{\mu\nu} - em_0^2 \right]. \quad (58)$$

Using energy  $\mathcal{E}$  and orbital momentum  $\mathcal{J}$  conservation and introducing the *proper* world-line time  $s$  ( $\frac{ds}{d\lambda} = em_0$ ), the “mass-shell” equation (the equation w.r.t. the “einbein”  $e$  produced by the action (58)) yields:

$$\left( \frac{d\eta}{ds} \right)^2 + \mathcal{V}_{\text{eff}}(\eta) = \frac{\mathcal{E}^2}{m_0^2}, \quad \mathcal{V}_{\text{eff}}(\eta) \equiv A(\eta) \left( 1 + \frac{\mathcal{J}^2}{m_0^2 C(\eta)} \right) \quad (59)$$

with  $\mathcal{A}(\eta)$ ,  $\mathcal{C}(\eta)$  – the same metric coefficients as in (48)–(50).

For generic values of the parameters the effective potential in the Bertotti-Robinson universe (48) (*i.e.*, for  $\eta < 0$ ) has harmonic-oscillator-type form. Therefore, a traveling observer starting in the Reissner-Nordström universe (49) (*i.e.*, at some  $\eta > 0$ ) and moving “radially” along the  $\eta$ -direction towards the horizon, will cross the wormhole “throat” ( $\eta = 0$ )

within finite interval of his/her proper time, then will continue into the Bertotti-Robinson universe subject to harmonic-oscillator deceleration force, will reverse back at the turning point and finally will cross the “throat” back into the Reissner-Nordström universe.

Let us stress that, as in the case of the previously constructed symmetric and asymmetric wormholes via *LL-branes* sitting on their “throats” [12, 13, 16, 17], the present Reissner-Nordström-to-Bertotti-Robinson wormhole is *not* traversable w.r.t. the “laboratory” time of a static observer in either universe.

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